Transportation Problem

Use the solver in **Excel** to find the number of units to ship from each factory to each customer that minimizes the total cost.

Formulate the Model

The model we are going to solve looks as follows in Excel.

	A	B	C	D	E	F	G	H	I	J
1	Transportation Problem									
2										
3		Unit Cost	Customer 1	Customer 2	Customer 3					
4		Factory 1	40	47	80					
5		Factory 2	72	36	58					
6		Factory 3	24	61	71					
7										
8										
9		Shipments	Customer 1	Customer 2	Customer 3		Total Out		Supply	
10		Factory 1	0	0	0		0	=	100	
11		Factory 2	0	0	0		0	=	200	
12		Factory 3	0	0	0		0	=	300	
13										
14		Total In	0	0	0					
15			=	=	=				Total Cost	
16		Demand	200	200	200				0	
17										

1. To formulate this **transportation problem**, answer the following three questions.

a. What are the decisions to be made? For this problem, we need Excel to find out how many units to ship from each factory to each customer.

b. What are the constraints on these decisions? Each factory has a fixed supply and each customer has a fixed demand.

c. What is the overall measure of performance for these decisions? The overall measure of performance is the total cost of the shipments, so the objective is to minimize this quantity.

2. To make the model easier to understand, create the following named ranges.

Range Name	Cells
UnitCost	C4:E6
Shipments	C10:E12
TotalIn	C14:E14
Demand	C16:E16
TotalOut	G10:G12

Supply	I10:I12
TotalCost	I16

3. Insert the following functions.

C	D	E	F	G	H	I
Customer 1	Customer 2	Customer 3				
40	47	80				
72	36	58				
24	61	71				
Customer 1	Customer 2	Customer 3		Total Out		Supply
0	0	0		=SUM(C10:E10)	=	100
0	0	0		=SUM(C11:E11)	=	200
0	0	0		=SUM(C12:E12)	=	300
=SUM(C10:C12)	=SUM(D10:D12)	=SUM(E10:E12)				
=	=	=				Total Cost
200	200	200				=SUMPRODUCT(UnitCost,Shipments)

Explanation: The SUM functions calculate the total shipped from each factory (Total Out) to each customer (Total In). Total Cost equals the **sumproduct** of UnitCost and Shipments.

Trial and Error

With this formulation, it becomes easy to analyze any trial solution.

For example, if we ship 100 units from Factory 1 to Customer 1, 200 units from Factory 2 to Customer 2, 100 units from Factory 3 to Customer 1 and 200 units from Factory 3 to Customer 3, Total Out equals Supply and Total In equals Demand. This solution has a total cost of 27800.

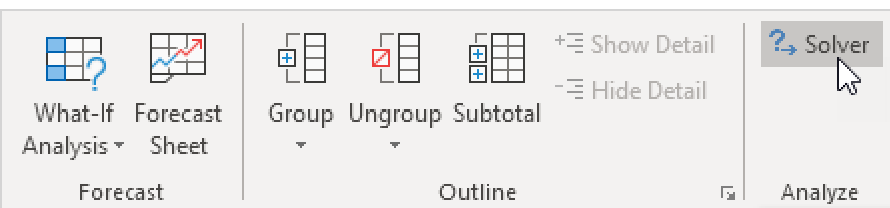
	A	B	C	D	E	F	G	H	I	J
1	Transportation Problem									
2										
3		Unit Cost	Customer 1	Customer 2	Customer 3					
4		Factory 1	40	47	80					
5		Factory 2	72	36	58					
6		Factory 3	24	61	71					
7										
8										
9		Shipments	Customer 1	Customer 2	Customer 3		Total Out		Supply	
10		Factory 1	100	0	0		100	=	100	
11		Factory 2	0	200	0		200	=	200	
12		Factory 3	100	0	200		300	=	300	
13										
14		Total In	200	200	200					
15			=	=	=				Total Cost	
16		Demand	200	200	200				27800	
17										

It is not necessary to use trial and error. We shall describe next how the **Excel Solver** can be used to quickly find the optimal solution.

Solve the Model

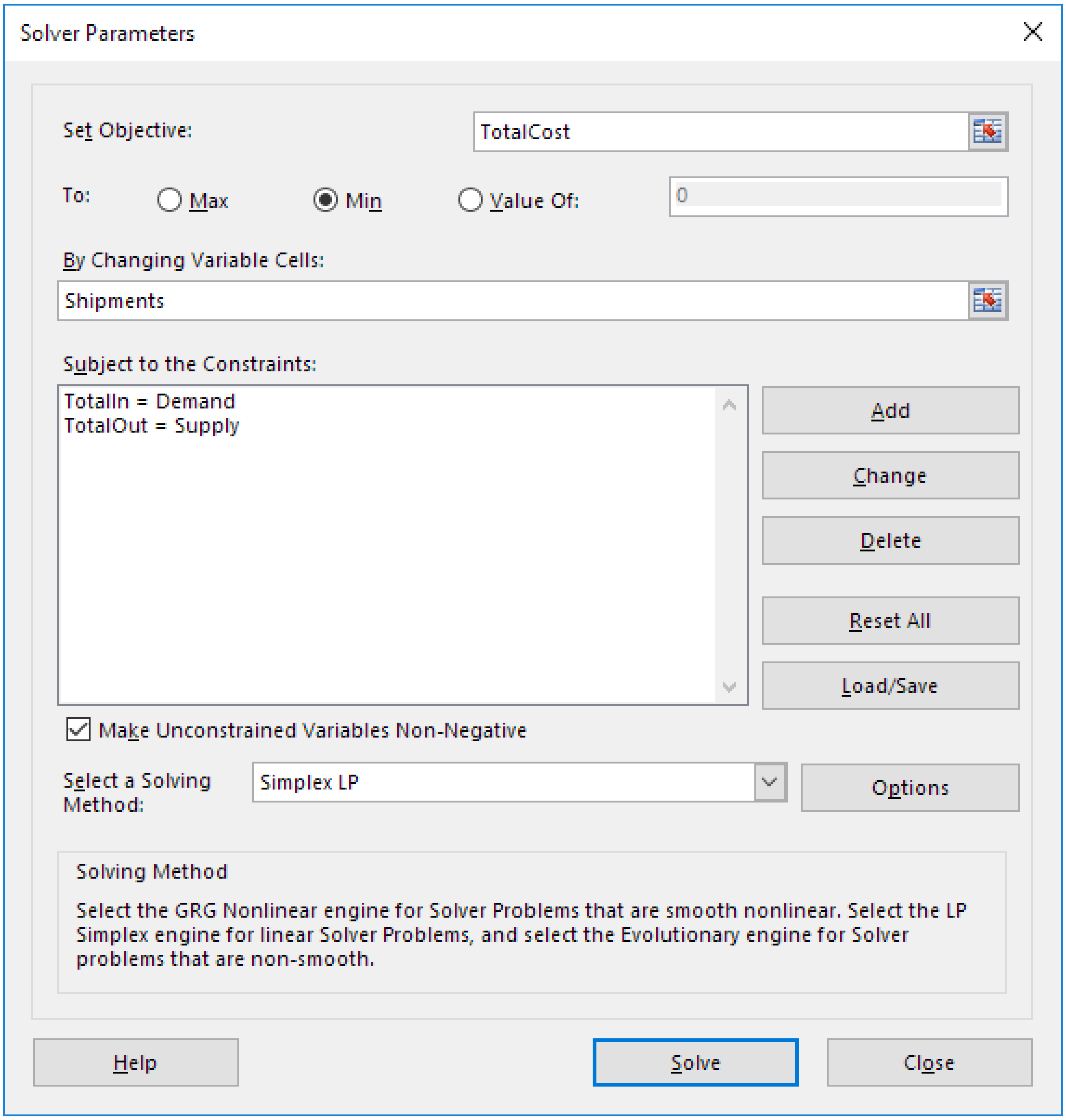
To find the optimal solution, execute the following steps.

1. On the Data tab, in the Analyze group, click Solver.



Note: can't find the Solver button? Click here to load the **Solver add-in**.

Enter the solver parameters (read on). The result should be consistent with the picture below.



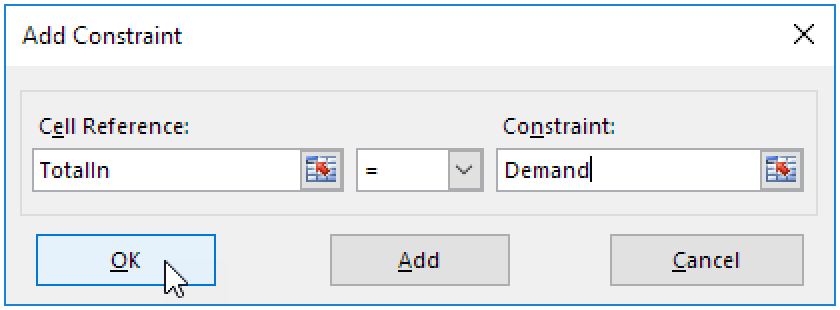
You have the choice of typing the range names or clicking on the cells in the spreadsheet.

2. Enter TotalCost for the Objective.

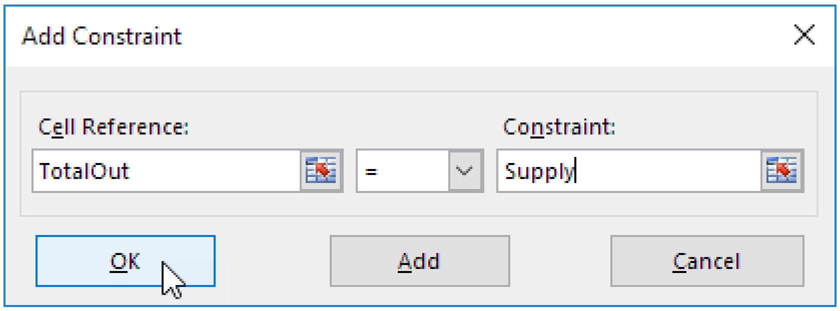
3. Click Min.

4. Enter Shipments for the Changing Variable Cells.

5. Click Add to enter the following constraint.



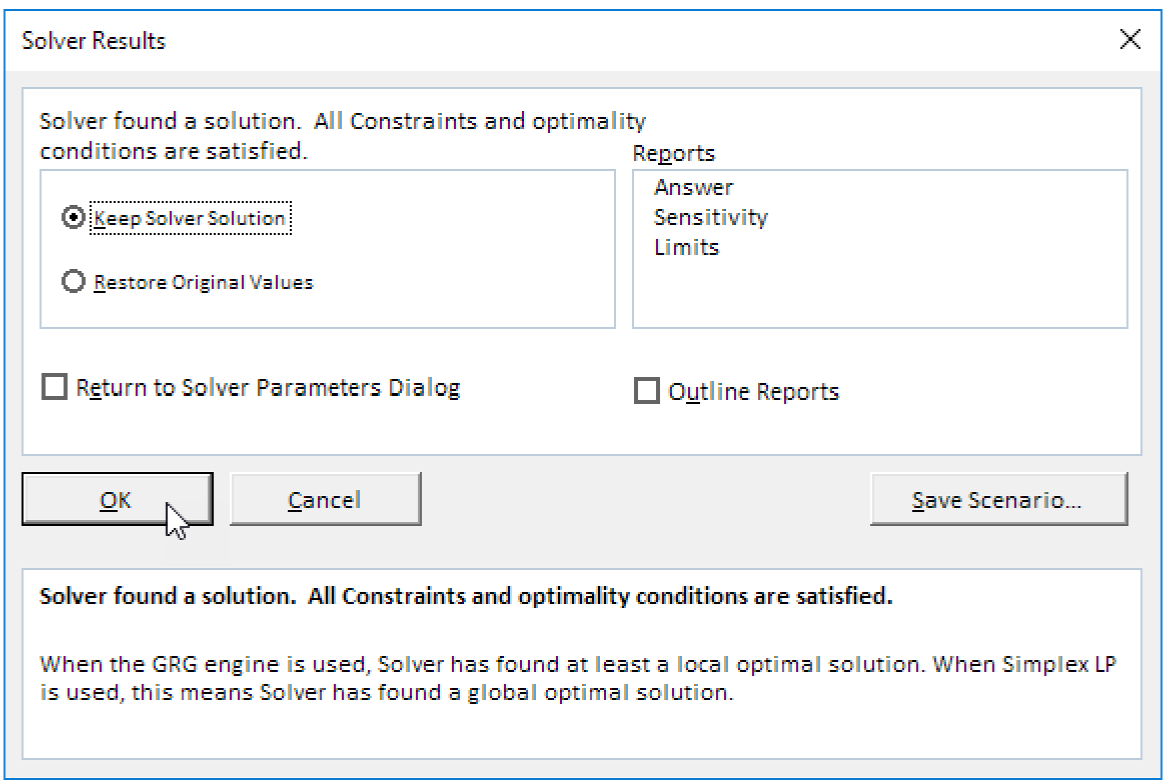
6. Click Add to enter the following constraint.



7. Check 'Make Unconstrained Variables Non-Negative' and select 'Simplex LP'.

8. Finally, click Solve.

Result:



The optimal solution:

Transportation Problem

Unit Cost	Customer 1	Customer 2	Customer 3
Factory 1	40	47	80
Factory 2	72	36	58
Factory 3	24	61	71

Shipments	Customer 1	Customer 2	Customer 3	Total Out		Supply
Factory 1	0	100	0	100	=	100
Factory 2	0	100	100	200	=	200
Factory 3	200	0	100	300	=	300
Total In	200	200	200			
	=	=	=			Total Cost
Demand	200	200	200			26000

Conclusion: it is optimal to ship 100 units from Factory 1 to Customer 2, 100 units from Factory 2 to Customer 2, 100 units from Factory 2 to Customer 3, 200 units from Factory 3 to Customer 1 and 100 units from Factory 3 to Customer 3. This solution gives the minimum cost of 26000. All constraints are satisfied.

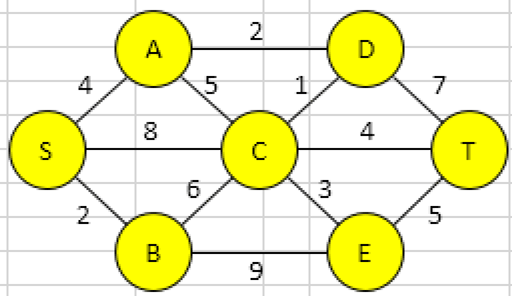
Shortest Path Problem

Use the solver in **Excel** to find the **shortest path** from node S to node T in an undirected network. Points in a network are called nodes (S, A, B, C, D, E and T). Lines in a network are called arcs (SA, SB, SC, AC, etc).

Formulate the Model

The model we are going to solve looks as follows in Excel.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Shortest Path Problem											
2												
3		From	To	Distance		Go		Nodes	Net Flow		Supply/Demand	
4		S	A	4		0		S	0	=	1	
5		S	B	2		0		A	0	=	0	
6		S	C	8		0		B	0	=	0	
7		A	C	5		0		C	0	=	0	
8		A	D	2		0		D	0	=	0	
9		B	C	6		0		E	0	=	0	
10		B	E	9		0		T	0	=	-1	
11		C	A	5		0						
12		C	B	6		0						
13		C	D	1		0						
14		C	E	3		0						
15		C	T	4		0						
16		D	A	2		0						
17		D	C	1		0						
18		D	T	7		0						
19		E	B	9		0						
20		E	C	3		0						
21		E	T	5		0						
22												
23				Total Distance		0						
24												



1. To formulate this **shortest path problem**, answer the following three questions.

a. What are the decisions to be made? For this problem, we need Excel to find out if an arc is on the shortest path or not (Yes=1, No=0). For example, if SB is part of the shortest path, cell F5 equals 1. If not, cell F5 equals 0.

b. What are the constraints on these decisions? The Net Flow (Flow Out - Flow In) of each node should be equal to Supply/Demand. Node S should only have one outgoing arc (Net Flow = 1). Node T should only have one ingoing arc (Net Flow = -1). All other nodes should have one outgoing arc and one ingoing arc if the node is on the shortest path (Net Flow = 0) or no flow (Net Flow = 0).

c. What is the overall measure of performance for these decisions? The overall measure of performance is the total distance of the shortest path, so the objective is to minimize this quantity.

2. To make the model easier to understand, create the following named ranges.

Range Name	**Cells**
From	B4:B21
To	C4:C21
Distance	D4:D21
Go	F4:F21
NetFlow	I4:I10
SupplyDemand	K4:K10
TotalDistance	F23

3. Insert the following functions.

F	G	H	I	J	K
Go		Nodes	Net Flow		Supply/Demand
0		S	=SUMIF(From,H4,Go)	=	1
0		A	=SUMIF(From,H5,Go)-SUMIF(To,H5,Go)	=	0
0		B	=SUMIF(From,H6,Go)-SUMIF(To,H6,Go)	=	0
0		C	=SUMIF(From,H7,Go)-SUMIF(To,H7,Go)	=	0
0		D	=SUMIF(From,H8,Go)-SUMIF(To,H8,Go)	=	0
0		E	=SUMIF(From,H9,Go)-SUMIF(To,H9,Go)	=	0
0		T	=-SUMIF(To,H10,Go)	=	-1
0					
0					
0					
0					
0					
0					
0					
0					
0					
0					
0					
=SUMPRODUCT(Distance,Go)					

Explanation: The **SUMIF** functions calculate the Net Flow of each node. For node S, the SUMIF function sums the values in the Go column with an "S" in the From column. As a result, only cell F4, F5 or F6 can be 1 (one outgoing arc). For node T, the SUMIF function sums the values in the Go column with a "T" in the To column. As a result, only cell F15, F18 or F21 can be 1 (one ingoing arc).

For all other nodes, Excel looks in the From and To column. Total Distance equals the **sumproduct** of Distance and Go.

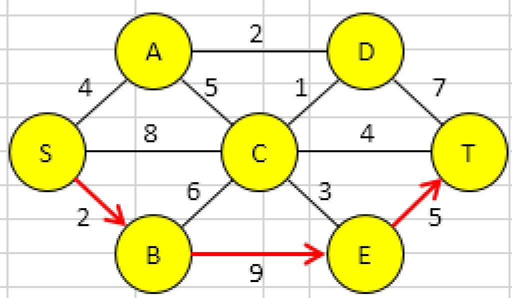
Trial and Error

With this formulation, it becomes easy to analyze any trial solution.

1. For example, the path SBET has a total distance of 16.

Shortest Path Problem

From	To	Distance	Go	Nodes	Net Flow		Supply/Demand
S	A	4	0	S	1	=	1
S	B	2	1	A	0	=	0
S	C	8	0	B	0	=	0
A	C	5	0	C	0	=	0
A	D	2	0	D	0	=	0
B	C	6	0	E	0	=	0
B	E	9	1	T	-1	=	-1
C	A	5	0				
C	B	6	0				
C	D	1	0				
C	E	3	0				
C	T	4	0				
D	A	2	0				
D	C	1	0				
D	T	7	0				
E	B	9	0				
E	C	3	0				
E	T	5	1				
		Total Distance	16				

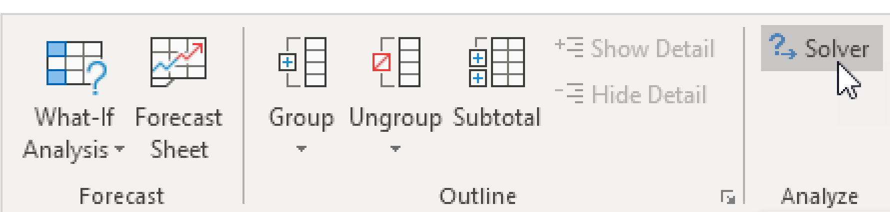


It is not necessary to use trial and error. We shall describe next how the **Excel Solver** can be used to quickly find the optimal solution.

Solve the Model

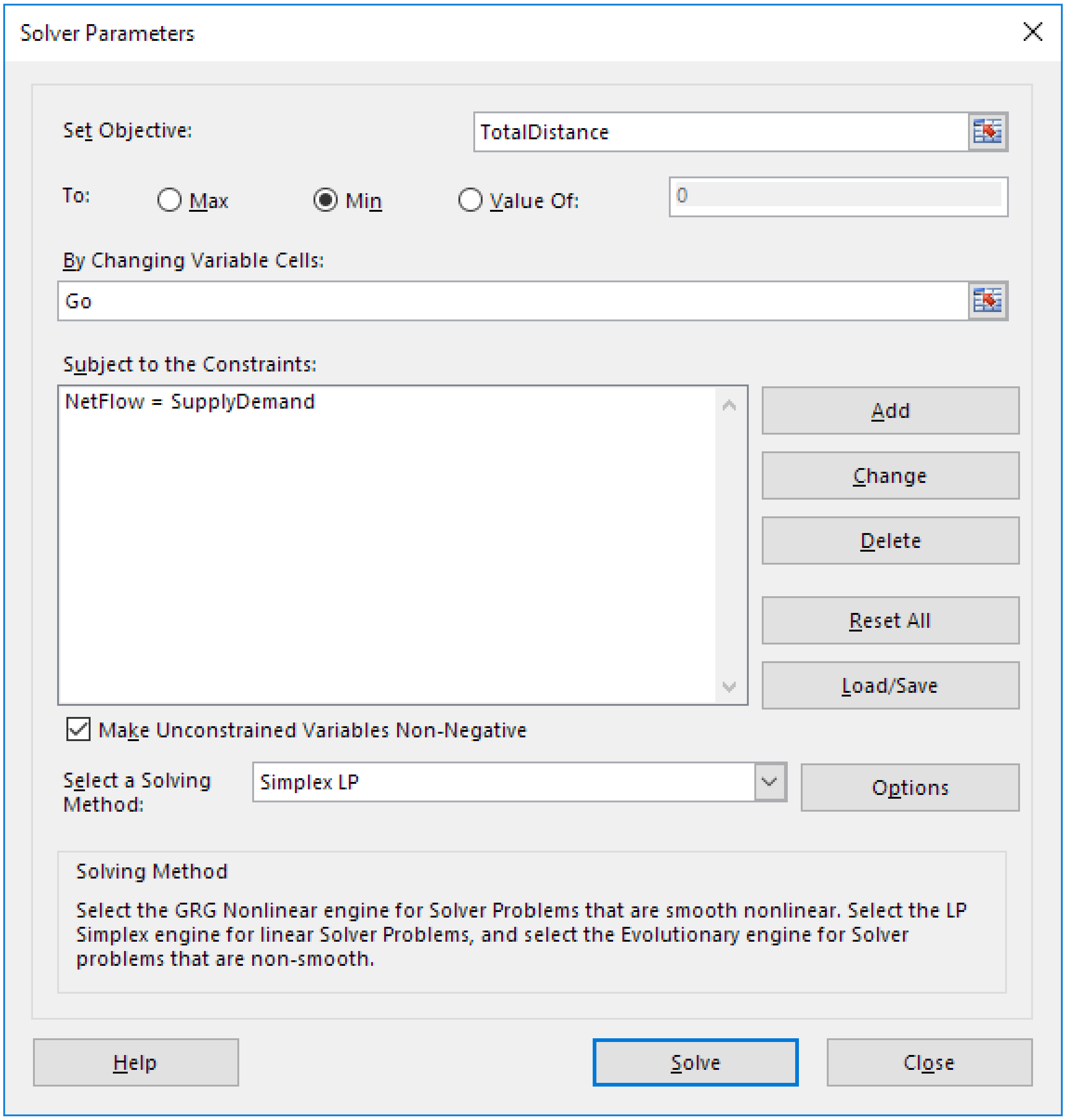
To find the optimal solution, execute the following steps.

1. On the Data tab, in the Analyze group, click Solver.



Note: can't find the Solver button? Click here to load the **Solver add-in**.

Enter the solver parameters (read on). The result should be consistent with the picture below.



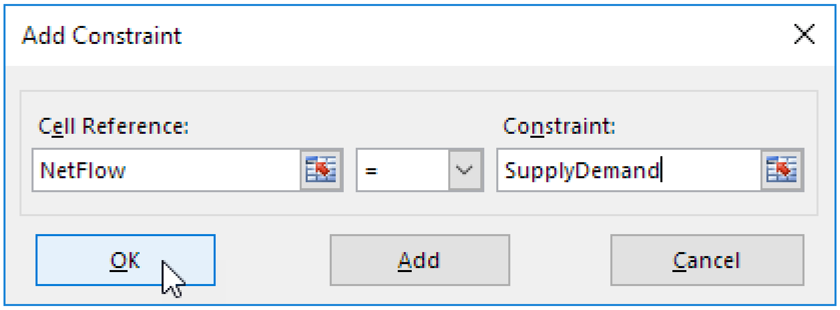
You have the choice of typing the range names or clicking on the cells in the spreadsheet.

2. Enter TotalDistance for the Objective.

3. Click Min.

4. Enter Go for the Changing Variable Cells.

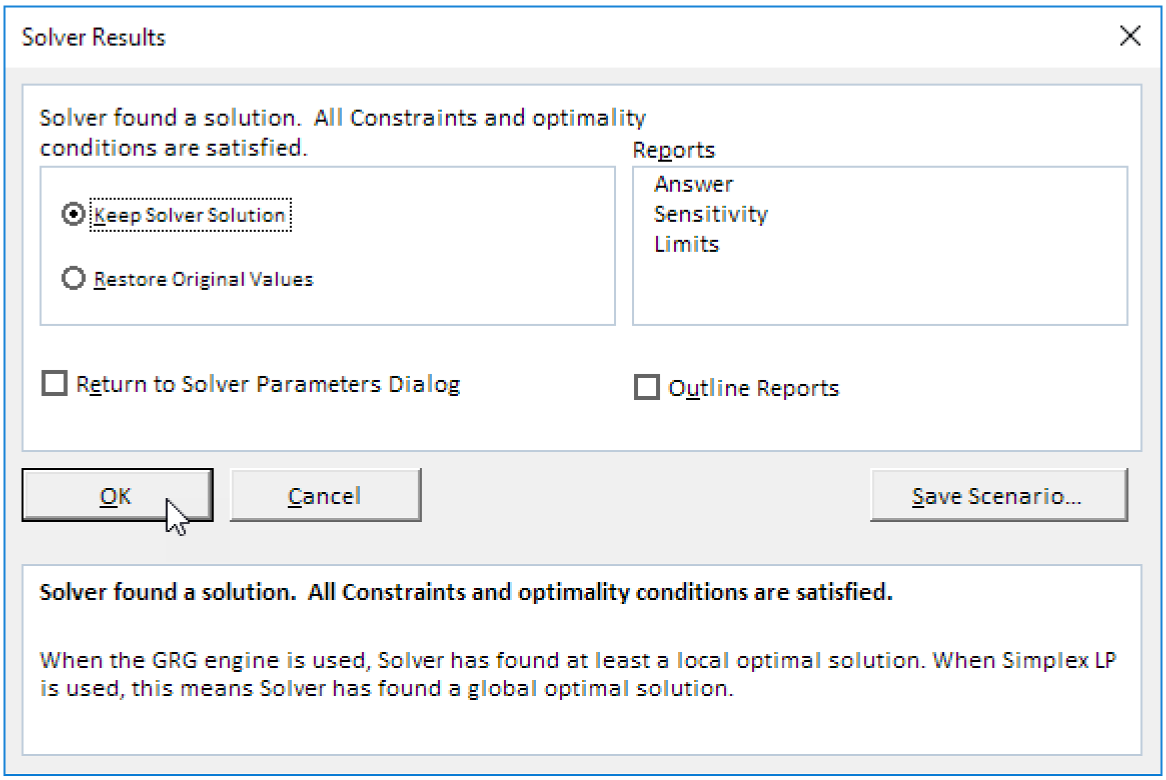
5. Click Add to enter the following constraint.



6. Check 'Make Unconstrained Variables Non-Negative' and select 'Simplex LP'.

7. Finally, click Solve.

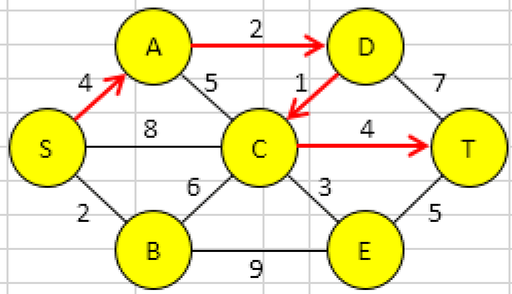
Result:



The optimal solution:

Shortest Path Problem

From	To	Distance	Go	Nodes	Net Flow		Supply/Demand
S	A	4	1	S	1	=	1
S	B	2	0	A	0	=	0
S	C	8	0	B	0	=	0
A	C	5	0	C	0	=	0
A	D	2	1	D	0	=	0
B	C	6	0	E	0	=	0
B	E	9	0	T	-1	=	-1
C	A	5	0				
C	B	6	0				
C	D	1	0				
C	E	3	0				
C	T	4	1				
D	A	2	0				
D	C	1	1				
D	T	7	0				
E	B	9	0				
E	C	3	0				
E	T	5	0				
	Total Distance		11				

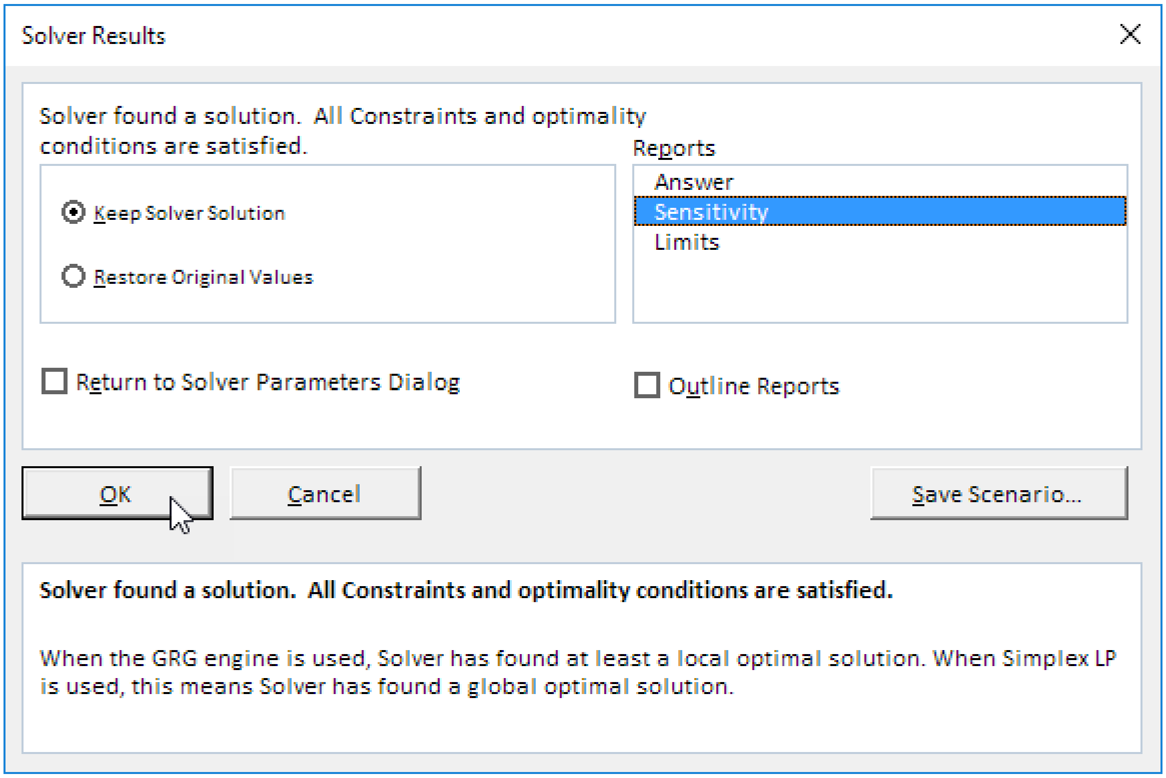


Conclusion: SADCT is the shortest path with a total distance of 11.

Sensitivity Analysis

Sensitivity analysis gives you insight in how the optimal solution changes when you change the coefficients of the model. After the solver found a solution, you can create a **sensitivity report**.

1. Before you click OK, select Sensitivity from the Reports section.



Below you can find the optimal solution and the sensitivity report.

	A	B	C	D	E	F	G	H	I	J
1	Cycle Trader									
2										
3			Bicycles	Mopeds	Child Seats					
4		Unit Profit	100	300	50					
5							Resources		Resources	
6							Used		Available	
7		Capital	300	1200	120		93000	≤	93000	
8		Storage	0.5	1	0.5		101	≤	101	
9										
10										
11			Bicycles	Mopeds	Child Seats				Total Profit	
12		Order Size	94	54	0				25600	
13										

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [sensitivity-analysis.xlsx]Sheet1
Report Created: 1/24/2020 10:34:47 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Order Size Bicycles	94	0	100	50	12.5
D12	Order Size Mopeds	54	0	300	66.66666667	100
E12	Order Size Child Seats	0	-20	50	20	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G7	Capital Used	93000	0.166666667	93000	28200	32400
G8	Storage Used	101	100	101	54	23.5

It is optimal to order 94 bicycles and 54 mopeds. This solution gives the maximum profit of 25600. This solution uses all the resources available (93000 units of capital and 101 units of storage). You can find these numbers in the Final Value column.

Reduced Cost

The reduced costs tell us how much the objective coefficients (unit profits) can be increased or decreased before the optimal solution changes. If we increase the unit profit of Child Seats with 20 or more units, the optimal solution changes.

1. At a unit profit of 69, it's still optimal to order 94 bicycles and 54 mopeds. Below you can find the optimal solution.

Cycle Trader

	Bicycles	Mopeds	Child Seats	Resources Used		Resources Available
Unit Profit	100	300	69			
Capital	300	1200	120	93000	≤	93000
Storage	0.5	1	0.5	101	≤	101

	Bicycles	Mopeds	Child Seats	Total Profit
Order Size	94	54	0	25600

2. At a unit profit of 71, the optimal solution changes.

	A	B	C	D	E	F	G	H	I	J
1	Cycle Trader									
2										
3			Bicycles	Mopeds	Child Seats					
4		Unit Profit	100	300	71					
5							Resources		Resources	
6							Used		Available	
7		Capital	300	1200	120		93000	≤	93000	
8		Storage	0.5	1	0.5		101	≤	101	
9										
10										
11			Bicycles	Mopeds	Child Seats				Total Profit	
12		Order Size	0	71.625	58.75				25658.75	
13										

Conclusion: it is only profitable to order child seats if you can sell them for at least 70 units.

Shadow Price

The shadow prices tell us how much the optimal solution can be increased or decreased if we change the right hand side values (resources available) with one unit.

1. With 101 units of storage available, the total profit is 25600. Below you can find the optimal solution.

	A	B	C	D	E	F	G	H	I	J
1	Cycle Trader									
2										
3			Bicycles	Mopeds	Child Seats					
4		Unit Profit	100	300	50					
5							Resources		Resources	
6							Used		Available	
7		Capital	300	1200	120		93000	≤	93000	
8		Storage	0.5	1	0.5		101	≤	101	
9										
10										
11			Bicycles	Mopeds	Child Seats				Total Profit	
12		Order Size	94	54	0				25600	
13										

2. With 102 units of storage available, the total profit is 25700 (+100).

	A	B	C	D	E	F	G	H	I	J
1	Cycle Trader									
2										
3			Bicycles	Mopeds	Child Seats					
4		Unit Profit	100	300	50					
5							Resources		Resources	
6							Used		Available	
7		Capital	300	1200	120		93000	≤	93000	
8		Storage	0.5	1	0.5		102	≤	102	
9										
10										
11			Bicycles	Mopeds	Child Seats				Total Profit	
12		Order Size	98	53	0				25700	
13										

Note: with a shadow price of 100 for this resource, this is according to our expectations. This shadow price is only valid between 101 - 23,5 and 101 + 54 (see sensitivity report).

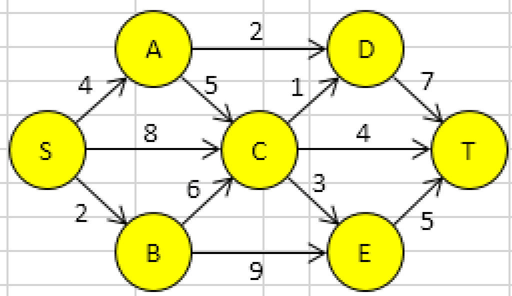
Maximum Flow Problem

Use the solver in **Excel** to find the **maximum flow** from node S to node T in a directed network. Points in a network are called nodes (S, A, B, C, D, E and T). Lines in a network are called arcs (SA, SB, SC, AC, etc).

Formulate the Model

The model we are going to solve looks as follows in Excel.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Maximum Flow Problem											
2												
3		From	To	Flow		Capacity		Nodes	Net Flow		Supply/Demand	
4		S	A	0	≤	4		S	0			
5		S	B	0	≤	2		A	0	=	0	
6		S	C	0	≤	8		B	0	=	0	
7		A	C	0	≤	5		C	0	=	0	
8		A	D	0	≤	2		D	0	=	0	
9		B	C	0	≤	6		E	0	=	0	
10		B	E	0	≤	9		T	0			
11		C	D	0	≤	1						
12		C	E	0	≤	3						
13		C	T	0	≤	4						
14		D	T	0	≤	7						
15		E	T	0	≤	5						
16												
17		Maximum Flow		0								
18												
19												
20												
21												



1. To formulate this **maximum flow problem**, answer the following three questions.

a. What are the decisions to be made? For this problem, we need Excel to find the flow on each arc. For example, if the flow on SB is 2, cell D5 equals 2.

b. What are the constraints on these decisions? The Net Flow (Flow Out - Flow In) of node A, B, C, D and E should be equal to 0. In other words, Flow Out = Flow In. Also, each arc has a fixed capacity. The flow on each arc should be less than this capacity.

c. What is the overall measure of performance for these decisions? The overall measure of performance is the maximum flow, so the objective is to maximize this quantity. The maximum flow equals the Flow Out of node S.

2. To make the model easier to understand, create the following **named ranges**.

Range Name	Cells
From	B4:B15
To	C4:C15
Flow	D4:D15
Capacity	F4:F15
SupplyDemand	K5:K9
MaximumFlow	D17

3. Insert the following functions.

D	E	F	G	H	I	J	K	L
Flow		Capacity		Nodes	Net Flow		Supply/Demand	
0	≤	4		S	=SUMIF(From,H4,Flow)			
0	≤	2		A	=SUMIF(From,H5,Flow)-SUMIF(To,H5,Flow)	=	0	
0	≤	8		B	=SUMIF(From,H6,Flow)-SUMIF(To,H6,Flow)	=	0	
0	≤	5		C	=SUMIF(From,H7,Flow)-SUMIF(To,H7,Flow)	=	0	
0	≤	2		D	=SUMIF(From,H8,Flow)-SUMIF(To,H8,Flow)	=	0	
0	≤	6		E	=SUMIF(From,H9,Flow)-SUMIF(To,H9,Flow)	=	0	
0	≤	9		T	=-SUMIF(To,H10,Flow)			
0	≤	1						
0	≤	3						
0	≤	4						
0	≤	7						
0	≤	5						
=I4								

Explanation: The **SUMIF** functions calculate the Net Flow of each node. For node A, the first SUMIF function sums the values in the Flow column with an "A" in the From column (Flow Out). The second SUMIF function sums the values in the Flow column with an "A" in the To column (Flow In). Maximum Flow equals the value in cell I4, which is the flow out of node S. Because node A, B, C, D and E have a Net Flow of 0, Flow Out of node S will equal Flow In of node T.

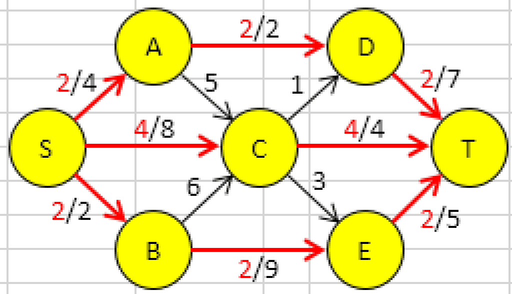
Trial and Error

With this formulation, it becomes easy to analyze any trial solution.

1. For example, the path SADT with a flow of 2. The path SCT with a flow of 4. The path SBET with a flow of 2. These paths give a total flow of 8.

Maximum Flow Problem

From	To	Flow		Capacity	Nodes	Net Flow		Supply/Demand
S	A	2	≤	4	S	8		
S	B	2	≤	2	A	0	=	0
S	C	4	≤	8	B	0	=	0
A	C	0	≤	5	C	0	=	0
A	D	2	≤	2	D	0	=	0
B	C	0	≤	6	E	0	=	0
B	E	2	≤	9	T	-8		
C	D	0	≤	1				
C	E	0	≤	3				
C	T	4	≤	4				
D	T	2	≤	7				
E	T	2	≤	5				
Maximum Flow		8						

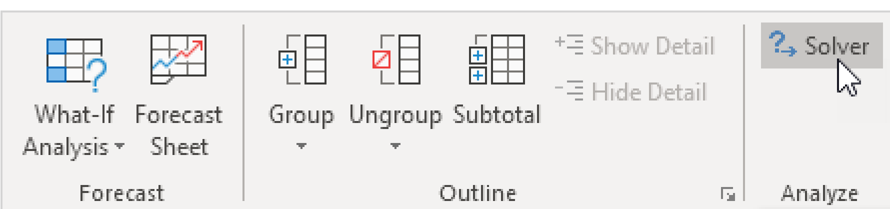


It is not necessary to use trial and error. We shall describe next how the **Excel Solver** can be used to quickly find the optimal solution.

Solve the Model

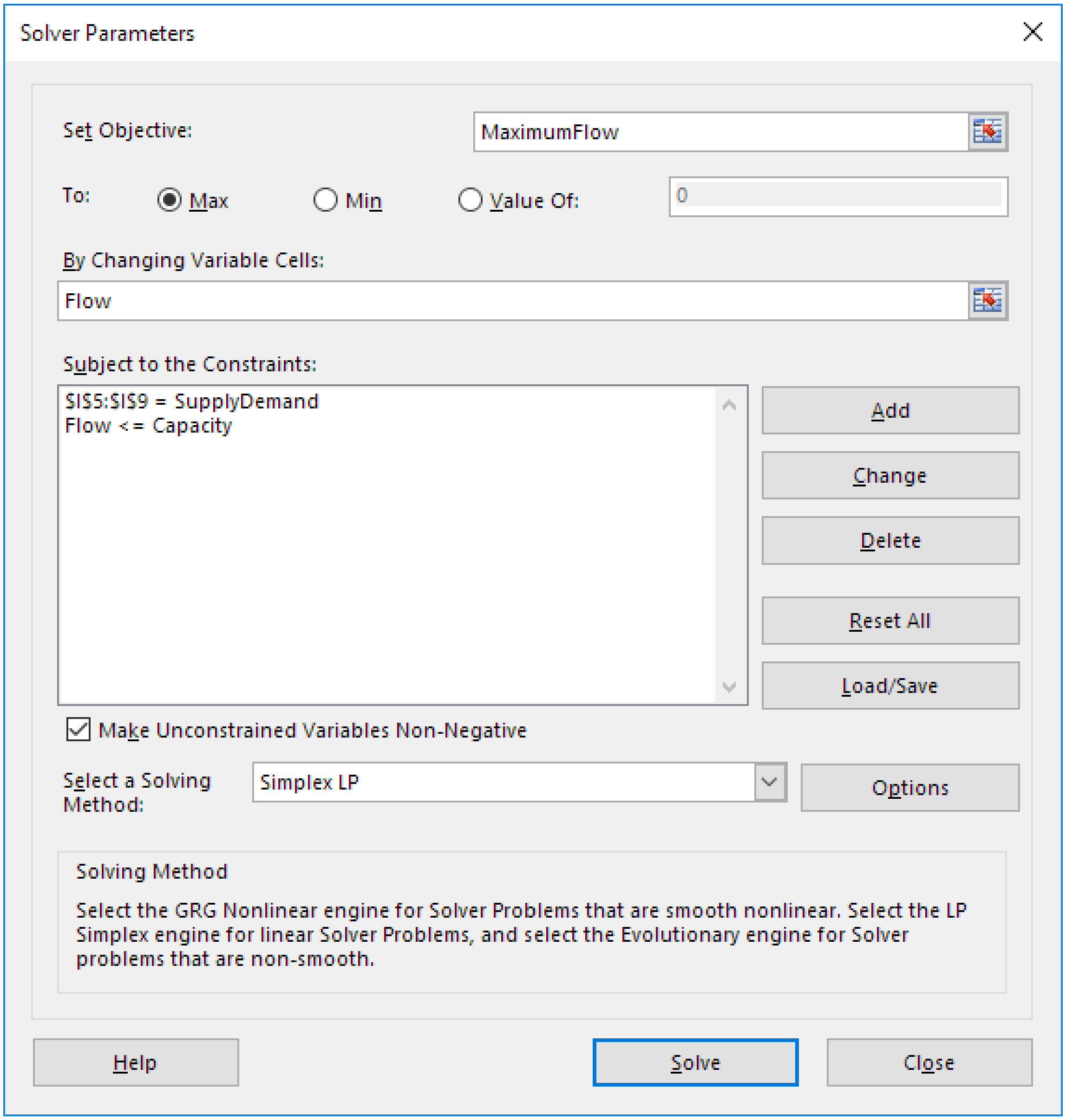
To find the optimal solution, execute the following steps.

1. On the Data tab, in the Analyze group, click Solver.



Note: can't find the Solver button? Click here to load the **Solver add-in.**

Enter the solver parameters (read on). The result should be consistent with the picture below.



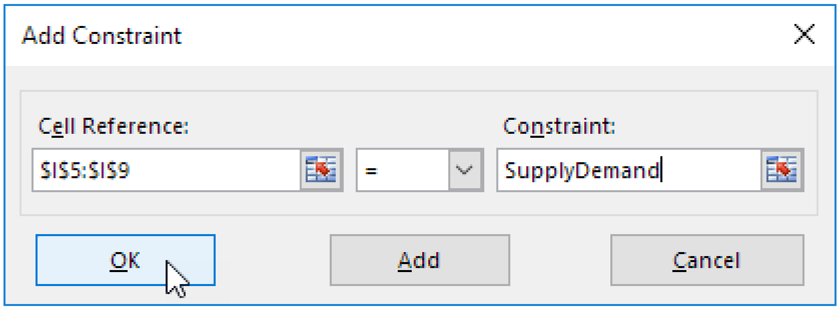
You have the choice of typing the range names or clicking on the cells in the spreadsheet.

2. Enter MaximumFlow for the Objective.

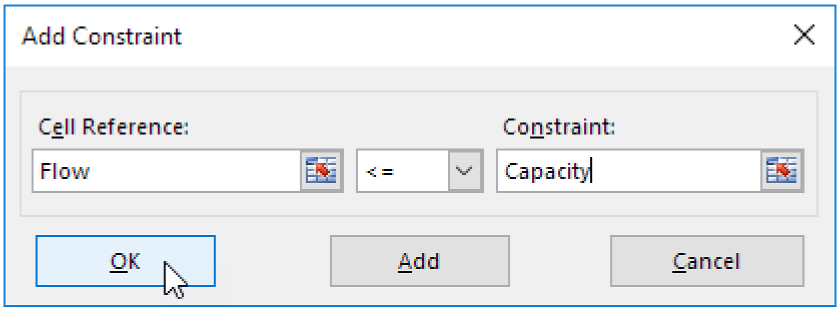
3. Click Max.

4. Enter Flow for the Changing Variable Cells.

5. Click Add to enter the following constraint.



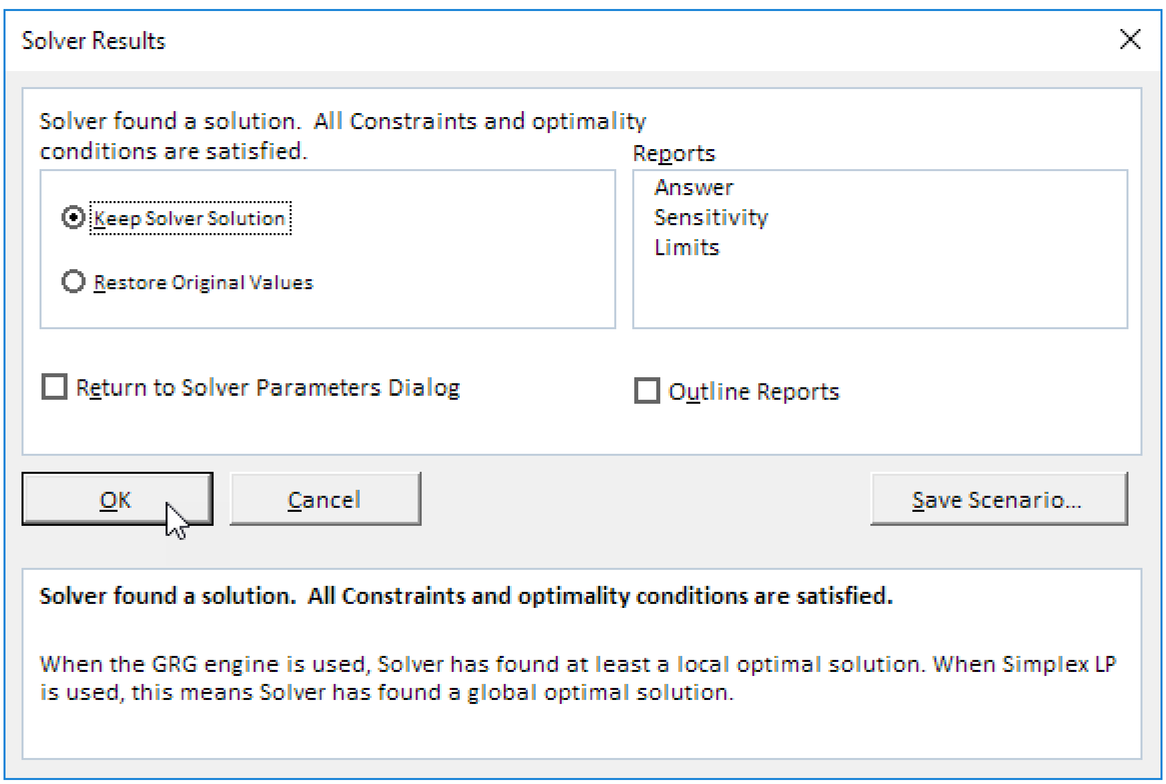
6. Click Add to enter the following constraint.



7. Check 'Make Unconstrained Variables Non-Negative' and select 'Simplex LP'.

8. Finally, click Solve.

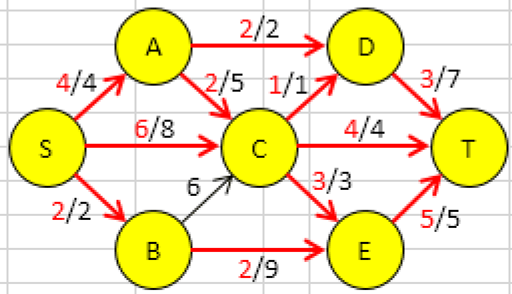
Result:



The optimal solution:

Maximum Flow Problem

From	To	Flow		Capacity	Nodes	Net Flow		Supply/Demand
S	A	4	≤	4	S	12		
S	B	2	≤	2	A	0	=	0
S	C	6	≤	8	B	0	=	0
A	C	2	≤	5	C	0	=	0
A	D	2	≤	2	D	0	=	0
B	C	0	≤	6	E	0	=	0
B	E	2	≤	9	T	-12		
C	D	1	≤	1				
C	E	3	≤	3				
C	T	4	≤	4				
D	T	3	≤	7				
E	T	5	≤	5				
Maximum Flow		12						



Conclusion: the path SADT with a flow of 2. The path SCT with a flow of 4. The path SBET with a flow of 2. The path SCET with a flow of 2. The path SACET with a flow of 1. The path SACDT with a flow of 1. These paths give a maximum flow of 12.